DERIVATIVES ON MARKET VOLATILITY: HEDGING TOOLS LONG OVERDUE

Robert E. Whaley

ROBERT E. WHALEY is the T. Austin Finch Foundation professor of business administration at the Fuqua School of Business at Duke University in Durham, North Carolina.
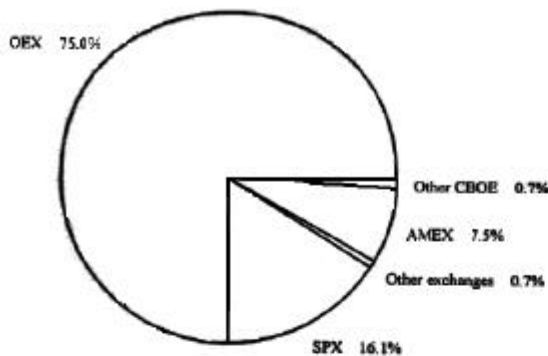
The Chicago Board Options Exchange Market Volatility Index, based on the implied volatilities of OEX options, provides not only a reliable estimate of short-term stock market volatility but also a volatility "standard" upon which futures and options contracts can be written. This article shows how volatility derivatives can be used to provide a simple, cost-effective means for hedging the market volatility risk of portfolios that contain options or securities with option-like features. Market volatility derivatives should prove to be valuable risk management tools for option market makers, portfolio insurers, and covered call writers.

The Chicago Board Options Exchange Market Volatility Index (ticker symbol VIX), which is based on the implied volatilities of eight different OEX option series, represents a market consensus forecast of stock market volatility over the next thirty calendar days. The Volatility Index can help the investment community in at least two important ways.

First, it provides a reliable estimate of expected short-term stock market volatility.[1] Expected market volatility is a critical piece of information to many investment decisions; the asset allocation decision is one. Second, it offers a market volatility "standard" upon which derivative contracts may be written. Such a standard must be based on a highly liquid underlying security market. In the case of VIX, the underlying security market is the OEX options market — by far the most active index option market in the U.S.

EXHIBIT 1
PROPORTION OF TOTAL NUMBER OF INDEX OPTIONS
TRADED IN THE U.S. IN 1992

Source: Chicago Board Options Exchange.

Exhibit 1 shows that OEX options accounted for 75% of the total number of index option contracts traded domestically in 1992. Average daily trading volume for OEX calls was 120,475, for puts 125,302.[2] Because it is based on the implied volatilities from the prices of highly active OEX options, VIX provides an *up-to-the-minute* account of new information affecting market volatility.

The purpose of this article is to show how futures and options contracts on the CBOE Market Volatility Index may be used to manage the volatility

EXHIBIT 2
LEVELS OF VIX AND OEX AT THE CLOSE OF TRADING
EACH WEDNESDAY — JANUARY 1988 THROUGH 1992

risk exposure of portfolios containing options or securities with option-like features. Following some basic description, we show how to measure a portfolio's volatility risk and how that risk exposure can be managed using exchange-traded index options as well as the volatility derivatives. Volatility derivatives are shown to be more effective and less expensive.

I. CBOE MARKET VOLATILITY INDEX

The CBOE Market Volatility Index is based upon the implied volatilities of eight near-the-money, nearby, and second nearby OEX options.[3] To maintain consistency in the composition of VIX, the Volatility Index is constructed so that, at any given time, it represents the implied volatility of a hypothetical at-the-money OEX option with thirty calendar days to expiration. (A detailed description of VIX construction appears in the appendix.)

The at-the-money distinction is important for two reasons. First, it means that the index is based on the most actively traded OEX option series, which are the at-the-money series. Second, it means that the volatility index moves approximately linearly with volatility-induced movements in the underlying OEX option prices.[4] Because there is almost no convexity in the relation between changes in implied volatility and changes in option prices for at-the-money options, the hedging effectiveness of volatility index derivatives is maximized.

The thirty-calendar day distinction is also important. Fleming, Ostdiek, and Whaley [1993] document that short-term OEX options tend to have higher implied volatilities than longer-term options. While the explanation for this relation is unclear,[5] maintaining a constant time to expiration minimizes the effect that this consideration may have on the Volatility Index level.

Exhibit 2 shows the levels of the CBOE Market Volatility Index at the close of trading each Wednesday during the past five years (January 1988 through December 1992). Also included in the figure is the level of the S&P 100 stock index (OEX). The figure is interesting in a number of respects.

First, note that VIX generally declined over the period. At the beginning of 1988, the market was coming off the October 1987 market crash when the
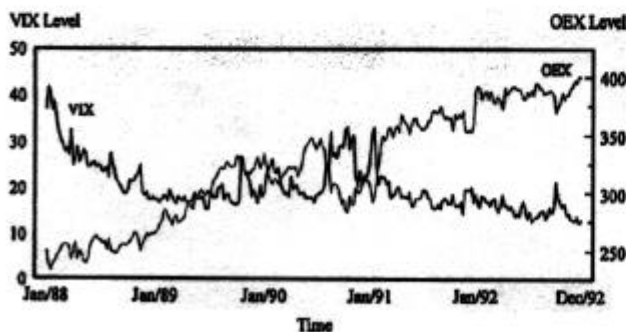
level of VIX exceeded 150%. As investors regained confidence in the future prospects of equities, the level of expected market volatility tapered off slowly. By the end of 1992, the level of VIX was below 15%.

Second, observe that expected market volatility experiences periodic jumps. These jumps are not without explanation. The jump in late 1989, for example, is the "mini-crash" resulting from the UAL restructuring failure. The jump in mid-1990 occurs with Iraq's invasion of Kuwait; the jump in early 1991 corresponds to U.N. forces attacking Iraq; and the jump in November 1992 reflects uncertainty about the U.S. presidential election.

Unexpected economic and political news causes investors to expect increased future volatility, and to bid up the prices of OEX options relative to the value of the underlying OEX index. VIX is merely reflecting the market's current thinking about expected volatility.

A third important behavior seen in Exhibit 2 is that VIX and OEX tend to move in opposite directions. This stands to reason. If expected market volatility increases, investors will demand a higher rate of return on stocks, and hence stock prices will fall.

The inverse correlation between VIX and OEX will not be perfect, however, because the volatility horizons in the two security valuations are different.[6] For stocks, the relevant expected volatility is the volatility over the life of the stock, which presumably is infinite. For options, it is generally short-term expected volatility that is important.

II. MANAGING VOLATILITY RISK EXPOSURE

For traders whose portfolios contain options or securities with option-like features, the two most important risk exposures are 1) what happens if the underlying security price changes unexpectedly, and 2) what happens if the expected volatility changes. We first show how portfolio managers can measure these risks by computing the delta and the vega of their option portfolios; we then demonstrate how these risks can be managed dynamically using derivative contracts. In particular, we show that volatility derivatives provide the most direct and inexpensive means of hedging volatility risk.

The analysis requires a number of simplifying assumptions. First, we assume that the option portfolio

consists entirely of European-style options written on the stock index portfolio. The stock index portfolio has a current level, S, a constant proportional dividend yield rate, δ, and a volatility rate, σ. The riskless rate of interest, r, is constant. All index option prices are assumed to obey the Merton [1973] constant proportional dividend yield option valuation formula.

Second, we assume that there exist futures contracts on the volatility of the stock index portfolio (i.e., on the "volatility index"), and that the volatility futures price F equals the underlying volatility index level.[7]

Third, we assume that there exist European-style volatility options on the volatility index. The volatility index has a current level $V(\equiv \sigma$, the volatility rate of the stock index portfolio as defined above) and a volatility rate σ_V. The prices of the volatility options are assumed to obey the Black [1976] futures option valuation formula.

In assuming that the option values follow the Merton [1973] and Black [1976] formulas, we are assuming implicitly that the stock index level and the volatility index level follow independent lognormal diffusion processes. All these assumptions can be relaxed in order to be more precise in measuring and managing volatility risk.

Measuring Risk Exposures

The most important determinants of option value are the underlying security price and the volatility rate. The effect of an unexpected change in the price of the underlying security on option value is measured by the option's delta.

A European-style call option on a stock index is valued by the equation,

$$c = e^{-\delta T}SN(d_1) - Xe^{-rT}N(d_2), \tag{1}$$

where

$$d_1 = \frac{\ln(S/X) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}},$$

and

$$d_2 = d_1 - \sigma\sqrt{T},$$

where X is the exercise price of the index option, T is the time to expiration, N is the cumulative normal probability, and all other notation is as previously defined. The change in the option value with respect to a change in the index level is therefore

$$\text{Delta}_c = e^{-\delta T}N(d_1) > 0. \qquad (2)$$

A European-style index put option is valued by the equation,

$$p = Xe^{-rT}N(-d_2) - e^{-\delta T}SN(-d_1), \qquad (3)$$

and has a delta of

$$\text{Delta}_p = -e^{-\delta T}N(-d_1) < 0. \qquad (4)$$

For portfolio risk management, individual option deltas are not as important as the delta of the overall portfolio. To compute the net delta of an index option portfolio, we multiply the delta of each option series by the number of contracts held, and then sum across all option series. That is,

Net portfolio delta =

$$\sum_{i=1}^{n} \text{Delta}_i \times \text{Number of contracts}_i, \qquad (5)$$

where n is the number of different option series in the portfolio. (For ease of exposition, we treat an option as applying to a single share. In practice, expressions such as (5) would have to be multiplied by the contract multiple, e.g., 100, in the case of OEX options.)

The effect of a change in the volatility parameter on option value is measured by the option's vega. If index call and put options are valued using (1) and (3), respectively, the vegas of the call and put options are equal and have the form,[8]

$$\text{Vega}_c = \text{Vega}_p = Se^{-\delta T}n(d_1)\sqrt{T} > 0, \qquad (6)$$

where $n(d_1)$ is the normal density function evaluated at d_1, that is,

$$n(d_1) \equiv \frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}.$$

Both the index call and put increase in value as volatility increases. This stands to reason, because an increase in volatility means that there is a greater probability of a large index move during the life of the option. The net vega of an index option portfolio is the sum of the weighted volatility exposures of the individual option series; that is,

Net portfolio vega =

$$\sum_{i=1}^{n} \text{Vega}_i \times \text{Number of contracts}_i \qquad (7)$$

An Illustration

The hedging problem in this illustration is that faced by an index option market maker who, in the course of business during a trading day, acquires a large option position and does not unwind the position by the close of trading. Holding the option position overnight exposes the market maker to unexpected overnight changes both in the index level and in volatility.[9]

To begin, we set the parameters of the problem and measure the market maker's portfolio risk exposures. Suppose that at the close of trading the market maker is holding the portfolio of short index option positions listed in Panel A of Exhibit 3. The fact that the market maker is short in all four option series means that net the market maker is short market volatility. (Recall that option value increases with the volatility rate.)

To measure the volatility risk exposure, we compute the net vega of the portfolio,

Net portfolio vega = 0.403(−50) + 0.642(−100)
+ 0.642(−75) + 0.642(−100) = −196.700.

This value implies that, if volatility increases by 100 basis points overnight,[10] the option portfolio value will fall by about $197, more than 4% of the portfolio value.

To measure the price risk exposure, we compute the net delta of the market maker's portfolio,

Net portfolio delta = 0.689(−50) + 0.530(−100)
− 0.465(−75) − 0.526(−100) = 0.025.

EXHIBIT 3

HYPOTHETICAL PORTFOLIO HELD BY INDEX OPTION MARKET MAKER AT THE CLOSE OF TRADING

Quantity	Call/Put	Exercise Price	Days to Expiration	Price	Delta	Vega	Theta
A. Unhedged Portfolio:							
-50	C	390	30	15.29	0.689	0.403	
-100	C	400	60	13.52	0.530	0.642	
-75	P	400	60	12.21	-0.465	0.642	
-100	P	405	60	14.84	-0.526	0.642	
Net Portfolio Position with Hedge				-4,516.25	0.025	-196.700	
B. Hedged Portfolio Using Index Put Option Only:							
448	P	395	30	6.52	-0.390	0.439	0.136
Total Hedge Position				2,920.96	-174.720	196.672	60.928
Net Portfolio Position with Hedge				-1,595.29	-174.695	-0.028	
C. Hedged Portfolio Using Index Call and Put Options:							
233	P	395	30	6.52	-0.390	0.439	0.136
209	C	405	30	7.18	0.436	0.451	0.158
Total Hedge Position				3,019.78	0.254	196.546	64.710
Net Portfolio Position with Hedge				-1,496.47	0.279	-0.154	
D. Hedged Portfolio Using Volatility Futures:							
197	F			20.00		1.000	
Total Hedge Position						197.000	
Net Portfolio Position with Hedge				-4,516.25	0.025	0.300	
E. Hedged Portfolio Using Volatility Call:							
364	C	20	30	1.71		0.541	0.028
Total Hedge Position				622.44		196.924	10.192
Net Portfolio Position with Hedge				-3,893.81	0.025	0.224	

Notes: The index portfolio's level is assumed to be 400, its volatility rate is 20%, and its dividend yield rate is 3%. The volatility index level is assumed to be 20%, and the volatility rate of the volatility index is 75%. The interest rate is 5%.

The portfolio is "delta-neutral." If the index level rises unexpectedly by a dollar, the portfolio will increase in value by only two cents.

### Hedging Volatility Risk Exposure Using Index Options

In the absence of market volatility derivatives, volatility risk can be hedged using index options. In the illustration, the market maker is short volatility. To negate this exposure, the market maker can buy either index calls or puts, since both calls and puts increase in

value with volatility. Since the portfolio's current net vega is -196.700, the net vega of the purchased index options must equal 196.700.

Suppose that an index put with an exercise price of 395 and a time to expiration of thirty days is available. Its price, delta, and vega are shown in Panel B of Exhibit 3. To eliminate volatility exposure using the 395 put, the market maker must buy

$$n_p = \frac{196.700}{0.439} \approx 448$$

EXHIBIT 4
PERCENT GAIN/LOSS IN VALUE OF PORTFOLIO FOR UNEXPECTED CHANGE IN MARKET VOLATILITY
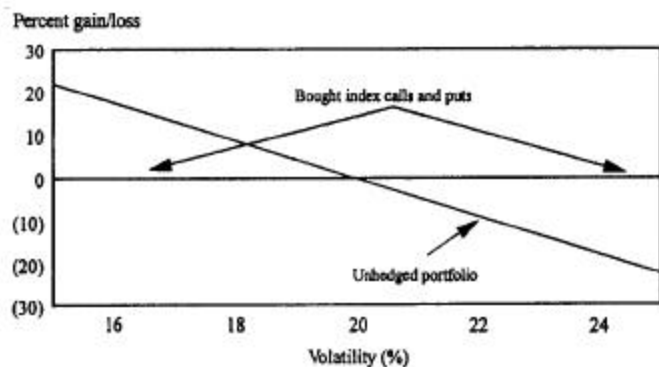
contracts. The cost of the options is $2,920.96. After the hedge is in place, the net vega of the portfolio is -0.028 so the market maker is now "vega-neutral."

Unfortunately, the purchase of a single index option series to hedge volatility risk has the undesired consequence of changing the market maker's delta exposure. After the puts are purchased, the option portfolio has a net delta of -174.695, as shown in Panel B. This means that if the stock index level increases by one point overnight, the portfolio value will drop by nearly 11%. In all likelihood, the market maker will find this delta exposure unacceptable.

To hedge volatility risk without changing delta risk, at least two index options must be used. The most natural way to hedge this risk is to use "volatility spreads." Volatility spreads consist of buying calls and puts in such a way that the incremental delta value is near zero. We assume that the market maker has the opportunity to buy not only the 395 put with thirty days to expiration but also a 405 call with thirty days to expiration. Panel C of Exhibit 3 shows both options.

To find the optimal number of calls and puts to buy, we must solve a simultaneous system of equations. First, since the unhedged portfolio is approximately delta-neutral, we want the net delta of the newly purchased options to be zero:

$$n_p Delta_p + n_c Delta_c = 0.$$

Second, since the unhedged portfolio has a net vega of -196.700, the net vega of the newly purchased calls and puts should be 196.700:

$$n_p Vega_p + n_c Vega_c = 196.700.$$

By solving this system of two equations with two unknowns, we find that $n_p \approx 233$ and that $n_c \approx 209$, as is shown in Panel C. The total cost of buying these options is $3,019.78. After the hedge is in place, the net delta exposure is 0.279, and the net vega exposure is -0.154, which means the portfolio, for all intents and purposes, is both delta-neutral and vega-neutral. The effective cost of using this hedge is the erosion in the options' value overnight.

The theta of an option is the change in option value with respect to a change in the option's remaining time to expiration. The theta values of the 395 put and the 405 call are also shown in Panel C. The net theta of the purchased index options is 64.710, which means that, holding other factors constant, the value of the hedge options will drop by approximately $64.71 overnight (i.e., one day for computation purposes).

To gauge the effectiveness of the volatility hedge, we compare the percentage change in the unhedged portfolio value to the percentage change in the hedge portfolio value when the volatility rate changes unexpectedly overnight.

Exhibit 4 shows that the unhedged portfolio has considerable volatility risk. If the volatility rate increases from 20% at the close of trading to 24% by the following morning, the portfolio value falls by more than 20%.

The hedged portfolio, on the other hand, is relatively immune to shifts in the volatility rate. Overnight shifts in the volatility rate as high as 500 basis points in either direction do not have an appreciable effect on portfolio value.[11]

Hedging Volatility Risk Exposure Using Volatility Futures

Hedging the market maker's vega risk exposure using volatility futures is straightforward. Since the net vega exposure of the market maker's portfolio is -196.700, the portfolio value will decrease by $196.700 for each 100 basis points of volatility

increase. Since the price of volatility futures moves directly with volatility, a 100-basis point increase in the volatility rate elicits a $1.00 increase in the futures price. Hence, the optimal number of volatility futures to buy is 197.

Panel D of Exhibit 3 shows the net effect of buying 197 volatility futures contracts. Since there is no cost in establishing the futures position, the portfolio value remains at $4,516.25. After the futures are purchased, the net vega of the portfolio is reduced to 0.300. The net delta of the portfolio, however, remains unchanged. Volatility derivatives, unlike index options, do not affect the delta exposure of the index option portfolio.

Exhibit 5 contrasts the percentage change of the unhedged portfolio with the percentage change of the hedged portfolio including the volatility futures as volatility changes. As the figure shows, the hedged portfolio value remains relatively constant over the 1,000-basis point volatility range considered.

Hedging Volatility Risk Exposure Using Volatility Options

Hedging the market maker's vega risk exposure using volatility options is nearly as straightforward as using volatility futures. The only difference is that the value of a volatility option does not move quite as quickly as volatility futures in response to volatility changes.

To quantify the rate of change, we need to compute the volatility option's delta. To do so, we apply the Black [1976] futures option valuation framework.[12] The valuation equation for a European-style call option on the volatility index is

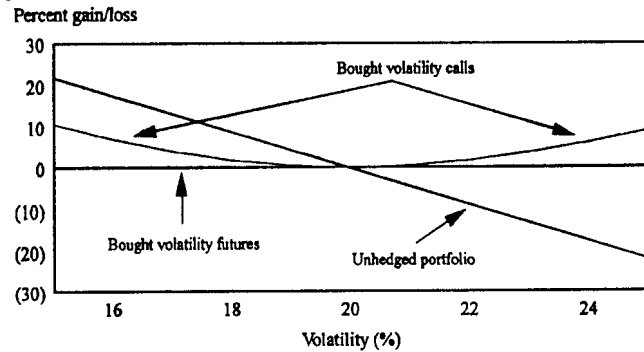$$c = e^{-rT}[VN(d_1) - XN(d_2)], \qquad (8)$$

where

$$d_1 = \frac{\ln(V/X) + 0.5\sigma_V^2 T}{\sigma_V \sqrt{T}}, \qquad (8A)$$

and

$$d_2 = d_1 - \sigma_V\sqrt{T}, \qquad (8B)$$

so the delta value of a volatility call option is

EXHIBIT 5
PERCENT GAIN/LOSS IN VALUE OF PORTFOLIO FOR UNEXPECTED CHANGE IN MARKET VOLATILITY

$$Delta_{c_v} = e^{-rT}N(d_1) > 0. \qquad (9)$$

The valuation equation for a European-style put option on the volatility index is

$$p = e^{-rT}[XN(-d_2) - VN(-d_1)],$$

with a delta value of

$$Delta_{p_v} = -e^{-rT}N(-d_1) < 0.$$

To set the volatility hedge for the market maker's portfolio, we simply divide the index option portfolio's volatility risk by the volatility option's delta, since the delta (rather than the vega) for a volatility option measures the option's sensitivity to volatility changes.

In the illustration, the unhedged portfolio has a net vega of -196.700. Suppose the market maker has the opportunity to buy volatility index calls with an exercise price of 20 and a time to expiration of thirty days. The optimal number of volatility calls to buy is

$$n_{c_v} = \frac{196.700}{0.541} \approx 36$$

Panel E of Exhibit 3 summarizes this hedge. The total cost of purchasing the volatility calls is

$622.44. With the purchase of the calls, the net portfolio vega is reduced to 0.224. Again, the net delta remains at 0.025 since volatility derivatives do not alter price risk exposure.

To gauge the effectiveness of the volatility option hedge relative to the volatility futures hedge, reconsider Exhibit 5. While the volatility futures hedge is immune to large shifts in volatility, the volatility option hedge provides an interesting convexity in which the market maker's profits increase in the event that volatility either rises or falls sharply. This convexity is not without cost, however. The volatility options are expected to erode in value by about $10.19 overnight.

Summary

Using volatility derivatives to manage market volatility risk offers at least two advantages over index option contracts. First, the hedge is simpler to implement. Using index options to hedge requires at least two different option series, while using volatility derivatives requires only one.

Second, volatility derivatives are less expensive. Using volatility futures, the hedge costs nothing. Using volatility options, the hedge costs only a small fraction of a similar hedge created using index options.[13]

In demonstrating the hedging mechanics of volatility derivatives, we made a number of simplifying assumptions regarding index option and volatility option valuation. Many of these assumptions can and should be relaxed in order to make option valuation and hence risk measurement more precise.

With respect to index option valuation, for example, the assumption that the dividends on the index portfolio are paid at a constant, proportional rate is unreliable.[14] The valuation methodology should account for discrete cash dividends on the underlying index.

In addition, OEX option contracts — the most active index options currently traded — are American-style, so the early exercise premium must be valued.[15] With respect to volatility option valuation, we assumed that the volatility options are European-style. If volatility options are American-style, other valuation methods must be used.[16]

We also made fundamental assumptions regarding the dynamics of stock index level and volatility index level movements. We assume that both the stock index and the volatility index follow separate lognormal diffusion processes and that the increments to the two processes are independent of each other. There is evidence instead that suggests market volatility is mean-reverting. There is also evidence that shows that stock index level movements and market volatility movements are inversely correlated. (See Exhibit 2.) Capturing these behaviors should lead to more precise option valuation,[17] more precise risk measurement, and hence greater hedging effectiveness using volatility index derivatives.

III. HEDGING INDIVIDUAL STOCK VOLATILITY

Market volatility derivatives can be used not only for index option portfolio volatility risk management but also for volatility risk management of any portfolio that has option-like securities whose values are sensitive to the economic climate in the stock market. We can demonstrate empirically that the movements in individual stock volatility are driven largely by movements in market volatility. Among other things, this implies that the volatility risk of options on individual stocks, as well as rights, warrants, and any convertible security issued by a firm, can be hedged using volatility derivatives.

To estimate the relation between stock volatility and market volatility, we collected return data for New York Stock Exchange and American Stock Exchange stocks from the Center for Research in Security Prices (CRSP) daily return file. Only stocks with continuous returns during the entire sixty-month period (January 1986 through December 1990) are included in the sample. The total number of stocks is 1,118.

Next, we compute the return standard deviation for each stock in each month using the stock's daily returns,

$$Vol_{i,t} = \sqrt{\frac{\sum_{t=1}^{T}(R_{i,t} - \bar{R}_i)^2}{T-1}},$$

where T is the number of days in the month. In addition, the return standard deviation of the S&P 100 index portfolio was computed each month.

Finally, to assess the degree to which changes in

individual stock volatility are explained by changes in market volatility, we regress monthly stock volatility changes, $\Delta Vol_{i,t}$, on monthly market volatility changes, $\Delta Vol_{m,t}$,

$$\Delta Vol_{i,t} = a + b\Delta Vol_{m,t} + e_{i,t},$$

for each stock in the sample.[18] A summary of the regression results is reported in Exhibit 6.

Exhibit 6 shows a number of interesting results. First, across all 1,118 stocks included in the sample, the average correlation between monthly stock volatility and market volatility changes is 0.644. This suggests that derivative contracts on market volatility can provide a reasonable, but not exceptional, hedge of individual stock volatility on average.

A problem with considering these composite results, however, is that there is considerable measurement error in the estimates of monthly volatility for the individual stocks, particularly for those stocks that do not trade frequently. (Recall that the estimates of volatility are based only on the daily returns of a single month — approximately twenty-two days.) The volatility estimates for inactive stocks will tend to be more noisy, undermining the estimated relation between individual stock volatility movements and movements in market volatility.

To examine the consequences of this bias, we consider only the 500 most active (i.e., highest trading volume) stocks. The regression results indicate that measurement error is indeed a problem. The average slope coefficient increases from 0.7991 in the full sample to 0.9210 for the 500 most active stocks, and the average correlation between the volatility changes also increases from 0.644 to 0.743.

Finally, we examine the estimated relation for selected stocks. During the sample period, IBM, XON, and GE had the greatest number of shares traded of any of the stocks in the sample. The results of their regressions are also included in Exhibit 6.

The degree of correlation between the stock volatility movements and market volatility movements for these stocks is extremely high. For IBM, the correlation is 0.949, for XON, 0.942, and for GE, 0.936. Among other things, this suggests that the volatility risk exposure of contingent claims on any of these stocks can be managed very effectively

using derivative contracts on the CBOE Market Volatility Index.

IV. SUMMARY

The marketplace will undoubtedly benefit from derivative contracts on the CBOE Market Volatility Index. Any portfolio consisting of options or securities with option-like features has volatility risk, which can be managed using volatility derivatives that are simpler and less expensive to implement than existing approaches.

To illustrate the use of volatility derivatives, we used the portfolio of an index option market maker. The job of a market maker is to stand ready to transact, which may involve accumulating a sizable option position. If this portfolio is held overnight, the market maker is exposed to volatility risk. With the advent of markets for volatility derivatives, this risk can be hedged, and market makers may pass on their cost savings to the investment public in the form of lower bid/ask spreads.

EXHIBIT 6
REGRESSION ESTIMATES OF CHANGES IN STOCK VOLATILITY ON CHANGES IN S&P 100 MARKET VOLATILITY

Sample	Statistic	$\hat{a}$	$t(\hat{a})$	$\hat{b}$	$t(\hat{b})$	Corr.
All Stocks	Mean	0.0029	0.09	0.7991	7.86	0.644
(n = 1, 118)	Median	0.0010	0.07	0.7681		7.11	0.686
Most Active						
Stocks	Mean	0.0017	0.08	0.9210	10.12	0.743
(n = 500)	Median	0.0008	0.06	0.9002	9.68	0.788
Individual Stocks						
IBM		-0.0015	-0.24	0.9440	22.64	0.949
XON		-0.0007	-0.08	1.1225	21.26	0.942
GE		-0.0004	-0.06	0.8624	20.11	0.936

Notes: Individual stock and market volatilities are computed each month using daily stock returns. The sample contains all NYSE and AMEX stocks that have continuous return data during the period January 1986 through December 1990. The regression equation is $\Delta Vol_{i,t} = a + b\Delta Vol_{m,t} + e_{i,t}$, where $\Delta Vol_{i,t}$ and $\Delta Vol_{m,t}$ are the changes in individual stock and S&P 100 market volatility, respectively.

Option market makers are not the only beneficiaries of volatility index derivative markets, however. Portfolio insurers can use short-term, exchange-traded index options to create synthetic long-term portfolio insurance, as long as they can buy volatility futures or volatility calls to lock in the level of market volatility when the short-term option portfolio is rolled over. Other users may be covered call option writers, who may face substantial losses if market volatility increases unexpectedly. Volatility derivatives would provide option writers an effective means of hedging their volatility risk exposure.

Nor is the set of beneficiaries limited to hedgers. Market research may produce predictions not only about the direction of the market movements but also about volatility of those movements. Volatility derivatives provide speculators with a means of trading volatility and profiting from superior market volatility prediction skills.

APPENDIX

COMPOSITION OF CBOE MARKET VOLATILITY INDEX (VIX)

The CBOE Market Volatility Index (VIX) is constructed from the implied volatilities of the eight near-the-money, nearby, and second nearby OEX option series. The implied volatilities are weighted in such a manner that VIX represents the implied volatility of a hypothetical thirty-calendar day (twenty-two-trading day), at-the-money OEX option. This appendix describes the VIX construction, beginning with the valuation method and data used to compute eight individual OEX option implied volatilities and ending with the algorithm used to combine the individual implied volatilities to generate the level of VIX.

IMPLIED VOLATILITY COMPUTATION

To compute an implied volatility, three types of information are required: 1) an option valuation model; 2) the values of the model's determinants (except for volatility); and 3) an observed option price. OEX option valuation is based on the Black-Scholes [1973]/Merton [1973] assumptions regarding the dynamics of security price movements. Because the OEX options are American-style, and because the underlying index portfolio pays discrete cash dividends, the option valuation problem is analytically intractable (i.e., no option valuation *equation* can be derived), and a valuation approximation is necessary. The approximation method used to compute the OEX option

implied volatilities is the cash-dividend-adjusted, Cox-Ross-Rubinstein [1979] binomial method described in Harvey and Whaley [1992].[19]

Second, the option model's determinants are the current index value, the option's exercise price and time to expiration, the riskless rate of interest, and the amount and timing of the anticipated cash dividends paid during the option's life. In generating the historical VIX series, the actual cash dividends paid during the option life are used as the proxy for anticipated dividends. The source of the dividend data is Harvey and Whaley [1992] prior to June 1988 and the *S&P 100 Information Bulletin* thereafter, through December 1992.

With VIX now computed on a real-time basis, the anticipated daily cash dividends of the S&P 100 index portfolio must be forecast. The dividend forecasts are obtained from a time sharing data service contracted by the CBOE.

The interest rate is the effective yield on a T-bill whose maturity most closely matches the option expiration, except where the option time to expiration is less than thirty days, in which case the T-bill with thirty days to maturity is used. The source of the T-bill bid/ask discounts used in the construction of the VIX historical series is *The Wall Street Journal*.

Of the remaining option determinants, exercise price and time to expiration are known. The reported OEX index level is used as a proxy for the current OEX index value. The reported index level is appended to each option transaction/quote record at the time the trade/quote is entered into the CBOE's Market Data Retrieval (MDR) system.

The distinction between "reported OEX index level" and "current OEX index value" is subtle but important. The reported OEX index level is computed throughout the trading day and is based on last transaction prices of the 100 index stocks. Since stocks do not trade continuously, reported levels of stock indexes, particularly broad-based indexes such as the S&P 100, are always "stale" indicators of actual index portfolio values.

When the market rises quickly during the trading day, for example, OEX option price movements lead movements in the reported OEX index because OEX options trade more frequently than does the "average" stock in the S&P 100 portfolio. This means that if an implied volatility is computed using an OEX call (put) price the implied volatility is upward- (downward-) biased because the reported OEX index is lower than its true (but as yet unobserved) value.[20] Since the upward (downward) bias of the call implied volatility is approximately equal to the downward (upward) bias of the put implied volatility, the effect of the infrequent trading of stocks in the index can be

and is mitigated within the VIX construction by averaging the call and put implied volatilities.

Third, the CBOE records both transaction and bid/ask price quotes in its MDR system. In both cases, the contemporaneous reported OEX index level is appended to the data record at the instant the option information is recorded. In constructing VIX, the midpoint of the bid/ask price quote is used as the option price in the implied volatility computation. This is done for two reasons.

First, bid/ask price quotes are entered instantly into the MDR system as they are heard by quote reporters stationed at various points in the OEX trading pit. OEX option transactions, on the other hand, are entered in different ways. Some transactions are entered manually when the trade ticket is received by a transaction reporter. For these transactions, there is a short delay in recording the option trade, so the trade price and the reported index level recorded on the transaction record are not synchronous.

Other transactions are entered electronically. The CBOE's Retail Automatic Execution System (RAES), for example, is available for certain option series. RAES automatically executes buy (sell) orders of ten contracts or fewer at the prevailing ask (bid) price. For these transaction records, the trade price and the index level are synchronous.

If the trade price and index level are not synchronous, implied volatilities based on transaction record information will have error. While the size of the error is probably small, its effect is unpredictable because the delay between the time the trade occurs and the time it is entered into the MDR system may differ across options (e.g., RAES versus non-RAES) and across varying levels of market activity.

Second, using the midpoint of the bid/ask price quotes eliminates the bouncing between bid and ask price levels (and hence computed implied volatilities) observed in the sequence of option transactions.[21]

Finally, it should be noted that VIX is based on trading days. If the time to expiration of the option is measured in calendar days, the implied volatility would be a volatility rate per calendar day. This means, among other things, that the return variance of the OEX index over a weekend (from Friday close to Monday close) should be three times greater than it is over any other pair of adjacent trading days during the week (say, Monday close to Tuesday close).

Empirically, this is simply not true. Volatility over the weekend is approximately the same as it is for other trading days.[22] For this reason, each (calendar-day) implied volatility rate is transformed to a trading-day basis in the following manner. First, according to the number of calendar days to expiration, N_c, the number of trading days, N_t,

is computed as

$$N_t = N_c - 2 \times int(N_c/7). \tag{A1}$$

An option with eight calendar days to expiration, for example, has six trading days to expiration.

Second, the implied volatility rate is multiplied by the ratio of the square root of the number of calendar days to the square root of the number of trading days, that is,

$$\sigma_t = \sigma_c \left(\frac{\sqrt{N_c}}{\sqrt{N_t}} \right),$$

where σ_t (σ_c) is the trading-day (calendar-day) implied volatility rate.[23,24]

INDEX CONSTRUCTION

The CBOE Market Volatility Index is constructed from the implied volatilities of the eight near-the-money, nearby, and second nearby OEX option series. The nearby OEX series are defined as the series with the shortest time to expiration but with at least eight calendar days to expiration.[25] The second nearby OEX series are the series of the adjacent contract month.

To explain the index construction, we denote the OEX option exercise price just below the current index level, S, as X_l and the exercise price just above the current index level as X_u. The implied volatilities of the nearby and second nearby OEX options are thus:

| | Nearby Contract (1) | | Second Nearby Contract (2) | |
	Call	Put	Call	Put
$X_l (< S)$	$\sigma_{c,1}^{X_l}$	$\sigma_{p,1}^{X_l}$	$\sigma_{c,2}^{X_l}$	$\sigma_{p,2}^{X_l}$
$X_u (\geq S)$	$\sigma_{c,1}^{X_u}$	$\sigma_{p,1}^{X_u}$	$\sigma_{c,2}^{X_u}$	$\sigma_{p,2}^{X_u}$

The first step in computation of the index level is to average the call and put implied volatilities in each of the four categories of options, that is,

$$\sigma_1^{X_l} = \left(\sigma_{c,1}^{X_l} + \sigma_{p,1}^{X_l} \right) / 2 \tag{A3A}$$

$$\sigma_2^{X_l} = \left(\sigma_{c,2}^{X_l} + \sigma_{p,2}^{X_l} \right) / 2$$

$$\sigma_1^{X_u} = \left(\sigma_{c,1}^{X_u} + \sigma_{p,1}^{X_u} \right) / 2 \tag{A3C}$$

$$\sigma_2^{X_u} = \left(\sigma_{c,2}^{X_u} + \sigma_{p,2}^{X_u} \right) / 2. \tag{A3D}$$

Recall that the averaging mitigates the effects that the staleness of the reported stock index level may have on computation of individual call and put option implied volatilities.

Next, interpolate between the nearby implied volatilities and the second nearby implied volatilities to create "at-the-money" implied volatilities. More specifically,

$$\sigma_1 = \sigma_1^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_1^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right) \qquad (A4A)$$

$$\sigma_2 = \sigma_2^{X_l}\left(\frac{X_u - S}{X_u - X_l}\right) + \sigma_2^{X_u}\left(\frac{S - X_l}{X_u - X_l}\right) \qquad (A4B)$$

Finally, interpolate (or, occasionally, extrapolate) between the nearby and second nearby implied volatilities to create a thirty-calendar day $(30 - 2 \times \text{int}(30/7) \equiv$ twenty-two-trading day) implied volatility. If N_{t_1} is the number of trading days to expiration of the nearby contract, and N_{t_2} is the number of trading days of the second nearby contract, the CBOE Market Volatility Index is

$$VIX = \sigma_1\left(\frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}}\right) + \sigma_2\left(\frac{22 - N_{t_1}}{N_{t_2} - N_{t_1}}\right). \qquad (A5)$$

ENDNOTES

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[1]In theory, the volatility implied by an OEX option price is an estimate of the expected volatility over the remaining life of the option. Empirical research assessing how well OEX implied volatility predicts future realized market volatility is only recently beginning to appear. See, for example, Canina and Figlewski [1993]; Fleming [1993]; and Fleming, Ostdiek, and Whaley [1993]. Fleming, Ostdiek, and Whaley find that the level of VIX is an accurate predictor of subsequently realized market volatility.

[2]These data were provided by Eileen Smith of the Chicago Board Options Exchange.

[3]Other types of volatility indexes have also been proposed. Brenner and Galai [1989], for example, argue that a volatility index could also be constructed from historical volatility or from some weighted combination of implied and historical volatility measures. Even if a combination predicts future realized volatility more accurately

than either measure by itself, including a historical volatility component in the volatility index will reduce the degree of correlation between changes in the volatility index and changes in the implied volatilities (or, equivalently, changes in option premiums), and thereby undermine the hedging effectiveness of volatility index derivatives.

[4]Feinstein [1989] shows that the Black-Scholes option valuation formula is approximately linear in volatility for at-the-money options.

[5]One possible explanation is that there is more speculation in the options markets when option time premiums are small. Another is that the option valuation model is misspecified.

[6]Indeed, with perfect negative correlation between the changes of VIX and OEX, volatility index derivatives would serve no purpose.

[7]That is, the cost of carrying the volatility index is assumed to be zero.

[8]Aside from the fact that vegas are equal for call and put options with the same terms, it is important to recognize that the option's sensitivity to volatility is greatest where the options are approximately at-the-money. Vega, as defined by (6), depends on the normal density $n(d_1)$. The normal density is maximized where $d_1 = 0$, which, according to (1A), happens where $S \approx X$.

[9]The dynamic hedge framework applied in this section is drawn from Stoll and Whaley [1993, Chapter 12].

[10]A basis point is 1/100th of 1%.

[11]The figure ignores the cost of the hedge.

[12]There is an implicit assumption that it is possible to create a riskless hedge between the volatility option and the underlying volatility index. While on first appearance the volatility index would seem to be a statistical artifact and not a traded asset, buying and selling the volatility index can be accomplished using the eight OEX options that constitute the volatility index. See the appendix for details of the index construction.

[13]In our illustration, the cost of the overnight hedge using volatility options was less than 16% (≡ 10.192/64.710) of the cost of using index options.

[14]Harvey and Whaley [1992] demonstrate how misleading an assumption of a constant dividend yield can be in valuing OEX options.

[15]Fleming and Whaley [1993] show how to value the interest income/cash dividend and wildcard early exercise features of the OEX options.

[16]To value American-style options on the volatility index under the assumption that the volatility index follows a lognormal diffusion process, the quadratic approximation in Whaley [1986] can be used.

[17]Hull and White [1987], Scott [1987], Wiggins

[1987], and Stein and Stein [1991] consider different models of stochastic volatility and their effects on option valuation. Ball [1993] reviews this work. Grunbichler and Longstaff [1993] discuss volatility option valuation where volatility is assumed to follow a mean-reverting process.

[18]A similar analysis could be performed by regressing the movements of stock option implied volatilities on movements in the CBOE Market Volatility Index.

[19]This valuation method accounts for the interest income/cash dividend motives for exercising OEX options early but does not account for the sequence of end-of-day wildcard options embedded in the OEX option contract. Fleming and Whaley [1993] show how to value the wildcard feature of OEX options. Although the wildcard privilege may contribute significantly to the overall option value, the wildcard premiums for the at-the-money call and put with the same expiration are approximately equal, and the size of the wildcard premium is approximately linear in time to expiration. This means that the Volatility Index, which is designed to be the implied volatility of a constant thirty-day, at-the-money OEX option, is slightly upward-biased (because the wildcard privilege is not valued), but is not influenced more heavily by calls or puts, nor does it change systematically through time as the times to expiration of the component options grow short.

[20]Ideally one would like to use the true OEX index value in the implied volatility computation. Since such an index value is unavailable, proxies must be considered. An ideal proxy would be the price of an actively traded futures contract on the OEX index, but the S&P 100 index futures market is now long defunct. Another possibility is the actively traded S&P 500 futures, although over long periods of time, the basis between the S&P 100 and S&P 500 cash indexes changes as small- (large-) capitalization stocks fall in and out of favor.

[21]Fleming, Ostdiek, and Whaley [1992] face this issue in examining the intraday price movements of OEX options and price movements of the underlying index portfolio stocks.

[22]Using all stocks on the New York Stock Exchange and the American Stock Exchange during the period 1963 through 1982, French and Roll [1986] estimate that weekend return variance is only 10.7% greater than trading day return variance. For the quintile of highest market capitalization stocks (which includes all of the S&P 100 stocks), the weekend return variance is only 8.2% higher.

[23]The logic underlying this transformation is that total volatility over the option's remaining life is the same, whether time to expiration is measured using calendar days or trading days. If time to expiration is measured in calendar days, the implied volatility is the volatility rate per calendar day, and total volatility over the option's remaining life is $\sigma_c \sqrt{N_c}$. If the volatility rate over the weekend is the same as for other trading days, the weekend must be treated like a trading day, and the volatility must be adjusted to a trading-day basis. Since we know total volatility over the life of the option is $\sigma_c \sqrt{N_c}$, we can find the volatility rate per trading day by setting the total volatility equal to $\sigma_t \sqrt{N_t}$ and solving for σ_t.

Note that this procedure is not the same as computing the implied volatility by simply inserting the number of trading days to expiration directly into the option valuation method. The option's time to expiration parameter affects valuation not only through total volatility (which, as has been argued, is best measured using trading days) but also through the expected rate of price appreciation in the index level over the option's life and through the length of time over which the option's expected cash is discounted to the present (both of which are more appropriately measured using calendar days).

[24]Fleming, Ostdiek, and Whaley [1993] show that this transformation to the volatility rate removes day-of-the-week seasonality in the OEX implied volatility computed on a calendar-day basis. Intuitively, the market values OEX options as if the volatility over the weekend is the same as for any other trading day. When one examines the movement of implied volatility (computed using calendar days) from Friday close to Monday close, the volatility rate increases, holding other factors constant, because the number of days to expiration in the option's life has been reduced by three instead of one.

[25]The volatility of implied OEX volatilities increases dramatically during the last week of trading. To avoid the spurious effect that this behavior would have on a volatility index, such options are not used.

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